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Fixed point theorems in ϵ-chainable fuzzy metric spaces via absorbing maps

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Abstract. In this paper, we prove a common fixed point by using a new notion of absorbing maps in ϵ-chainable fuzzy metric space with reciprocal continuity and semi-compatible maps. Also we illustrate the properties of absorbing maps. Moreover, we demonstrate the necessity of absorbing maps to find a common fixed point in ϵ-chainable fuzzy metric spaces. Our result generalizes and extend the results of Cho et al. [1] and many other similar results too.

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1. Introduction

In 1965 Zadeh [16] introduced the notion of fuzzy sets. After this during the last few decades many authors have establish the existence of a lots of fixed point theorems in fuzzy setting ; Especially Deng zi-ke [4], Erceg [5], George and Veeramani [7, 8], Kaleva and Seikkala [9], Kramosil and Michalek [10]. In [7] George and Veeramani modified the concept of fuzzy metric space which introduced by Kramosil and Michalek [10]. Cho et al. [3] introduced the notion of semi-compatible maps in a d-topological space. Singh et al. [13] introduced the notion of semi-compatible maps in fuzzy metric space, and prove a common fixed point theorem in this space. In [15] Vasuki introduce the concept of R-weakly commuting map, and prove a fixed point theorem in fuzzy metric space. The first author Ranadive et al [12] introduced the concept of absorbing mapping in metric space and prove the common fixed point theorem in this space. Moreover they observe that the new notion of absorbing map is neither a subclass of compatible maps nor a subclass of non-compatible maps. In

[11] Mishra et al introduced absorbing maps in fuzzy metric space. In this paper, we obtain common fixed point theorem by absorbing maps in ϵ-chainable fuzzy metric space with reciprocal continuity and semi-compatible maps.

2. Preliminaries

In this section we recall some definitions and known results in fuzzy metric space.

Definition 2.1. (Zadeh [16]) Let X be a non-empty set. A fuzzy set A in X is a function with domain X and value in $[0, 1]$.

Definition 2.2. (Schweizer and Sklar [14]) A triangular norm $*$ (shortly $t - norm$) is a binary operation on the unit interval $[0, 1]$ such that for all $a, b, c, d \in [0, 1]$ the following conditions are satisfied :

(1) $a * 1 = 1$;

(2) $a * b = b * a$;

(3) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$;

(4) $a * (b * c) = (a * b) * c$.

Definition 2.3. (George and Veeramani [7]) The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary non-empty set, $*$ is a continuous t_{norm} and M is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$ and $s, t > 0$.

(FM1) $M(x, y, 0) > 0$;

(FM2) $M(x, y, t) = 1$ for all $t > 0$, iff $x = y$;

(FM3) $M(x, y, t) = M(y, x, t)$;

(FM4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

(FM5) $M(x, y, .) : (0, \infty) \to [0, 1]$ is continuous.

Example 2.4. (George and Veeramani [7]) Let (X, d) be a metric space. Define $a * b = ab$ (or $a * b = \min[a, b]$) and for all $x, y \in X$ and $t > 0$, $M(x, y, t) = \frac{t}{t+d(x,y)}$. Then $M(X, M, *)$ is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition 2.5. (George and Veeramani [7] A sequence $\{x_n\}$ in a fuzzy metric space $M(X, M, *)$ is called Cauchy if for each $\epsilon > 0$ and $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$. A fuzzy metric space $M(X, M, *)$ is said to complete if every Cauchy sequence in X converge to a point in X. A sequence $\{x_n\}$ in X is convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) > 1 - \epsilon$ for each $t > 0$, there exists $n_0 \in N$.

Definition 2.6. A pair (A, B) of self maps of a fuzzy metric space $M(x, y, *)$ is said to be reciprocal continuous if $\lim_{n\to\infty} ABx_n = Ax$ and $\lim_{n\to\infty} BAx_n = Bx$, whenever there exists a sequence $x \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$. If A and B are both continuous then they are obviously reciprocally continuous but not converse need not be true.

Definition 2.7. (Cho and Kim) [2] Self mappings A and B of a fuzzy metric space $(X, M, *)$ is said to be weakly compatible if $ABx = BAx$ when $Ax = Bx$ for some $x \in X$.

Definition 2.8. A pair (A, B) of self-maps of a fuzzy metric space $(X, M, *)$ is said to be semi-compatible if $\lim_{n\to\infty} ABx_n = Bx$ whenever there exist a sequence $x_n \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Definition 2.9. (Cho and Jung) [1] Let $(X, M, *)$ be a fuzzy metric space and $\epsilon > 0$. A finite sequence $x = x_0, x_1, ..., x_n = y$ is called ϵ-chain from x to y if $M(x_i, x_{i-1}, t) > 1 - \epsilon$ for all $t > 0$ and $i = 1, 2, ..., n$.
A fuzzy metric space $(X, M, *)$ is called ϵ-chainable if for any $x, y \in X$, there exists an ϵ-chain from x to y.

Lemma 2.10. If for all $x, y \in X$, $t > 0$ and $0 < k < 1$, $M(x, y, kt) \geq M(x, y, t)$, then $x = y$.

Lemma 2.11. (Grabiec [6]) $M(x, y, .)$ is non-decreasing for all $x, y \in X$.

The following proposition show that in the concept of reciprocal continuity the notion of compatible and semi-compatibility of maps becomes equivalent.

Proposition 2.12. ([11] Mishra et al) Let A and B be two self maps on a fuzzy metric space $M(X, M, *)$. Assume that (A, B) is reciprocal continuous then (A, B) is semi-compatible if and only if (A, B) is compatible.

Definition 2.13. Let f and g be two self-maps on a fuzzy metric space $(X, M, *)$ then f is called g-absorbing if there exists a positive integer $R > 0$ such that $M(gx, gfx, t) \geq M(gx, fx, \frac{t}{R})$ for all $x \in X$. Similarly g is called f-absorbing if there exists a positive integer $R > 0$ such that $M(fx, fgx, t) \geq M(fx, gx, \frac{t}{R})$ for all $x \in X$.

Example 2.14. (George and Veeramani [7]) Let (X, d) be usual metric space where $X = [2, 20]$ and M be the usual fuzzy metric on $(X, M, *)$ where $* = t_{\min}$ be the induced fuzzy metric space with $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $M(x, y, 0) = 0$ for $x, y \in X$, $t > 0$. We define

$$fx = \begin{cases} 6 & if \;\; 2 \leq x \leq 5; \quad and \quad x = 6 \\ 10 & if \quad x > 6 \\ \frac{x-1}{2} & if \quad x \in (5, 6) \end{cases}$$

$$gx = \begin{cases} 2 & if \quad 2 \leq x \leq 5 \\ \frac{x+1}{3} & if \quad x > 5 \end{cases}$$

It is easy to see that both (f, g) and (g, f) are not compatible but f is $g-absorbing$ and g is $f-absorbing$. [Hint : Choose $x_n = 5 + \frac{1}{2n} : x \in N$][See [11]].

Example 2.15. ([11] Mishra et al) If $X = [0, 1]$ be a metric space and d and M are same as above example 2.12. Define $f, g : X \to X$ by $fx = \frac{x}{16}$ and $gx = 1-\frac{x}{3}$. In this example we can see that f and g are compatible pair of maps and f is g-absorbing while g is f-absorbing [Hint : range of $f = [0, \frac{1}{16}]$ and range of $g = [\frac{2}{3}, 1]$

Our next example to show that absorbing maps need not commute at their coincidence points, thus the notion of absorbing maps is different from other generalization of commutativity which force the mapping to commute at coincidence points.

Example 2.16. ([11] Mishra et al) Let $X = [0, 1]$ be a metric space and d and M are same as in above example 2.12. Define $f, g : X \to X$ by

$$fx = \begin{cases} 1 & for \quad x \neq 1 \\ 0 & for \quad x = 1 \end{cases}$$

and $gx = 1$ for $x \in X$.Then the maps f and g-absorbing for any $R > 1$ but the pair of maps (f, g) do not commute at their coincidence point $x = 0$.

Following theorem is proved by Cho et al [1]

Theorem 2.17. Let $(X, M, *)$ be a complete ϵ-chainable fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions;

(1) $AX \subset TX$ and $BX \subset SX$;

(2) A and S are continuous ;

(3) the pair $[A, S]$ and $[B, T]$ are weakly compatible ;

(4) there exists $q \in (0, 1)$ such that
$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$ for every $x, y \in X$ and $t > 0$. Then A, B, S and T have a unique common fixed point in X.

3. Main result

In this paper, we prove a fixed point theorem in which we totally replace continuity condition by using weaker notion of reciprocal continuity and employing absorbing mapping and semi-compatibility.

Theorem 3.1. Let A, B, S, T, L and M be self mappings of a complete ϵ-chainable fuzzy metric space $(X, M, *)$ with continuous $t-norm$ defined by $a * b = \min\{a, b\}$, satisfying :

(1) $L(X) \subseteq ST(X), M(X) \subseteq AB(X)$;

(2) M is ST absorbing ;

(3) $AB = BA$, $ST = TS$, $LB = BL$, $MT = TM$;

(4) there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t), M(Lx, STy, t)\}.$$

If $\{L, AB\}$ is reciprocally continuous semi-compatible maps. Then A, B, S, T, L and M have a unique fixed point in X.

Proof. Let $x_0 \in X$ be any arbitrary point. From (1), there exists $x_1, x_2 \in X$ such that $Lx_0 = STx_1 = y_0$ and $Mx_1 = ABx_2 = y_1$. Inductively we can construct sequence $\{x_n\}$ and $\{y_n\}$ in X such that $Lx_{2n} = STx_{2n+1} = y_{2n}$ and $Mx_{2n+1} =$

$ABx_{2n+2} = y_{2n+1}$ for $n = 0, 1, 2, ...$ Putting $x = x_{2n}, y = x_{2n+1}$ for $t > 0$ in (4); we get

$$M(Lx_{2n}, Mx_{2n+1}, kt) \geq \min\{M(ABx_{2n}, STx_{2n+1}, t), M(Lx_{2n}, ABx_{2n}, t), M(Mx_{2n+1}, STx_{2n+1}, t), M(Lx_{2n}, STx_{2n+1}, t)\}$$

that is,

$$\begin{aligned} M(y_{2n}, y_{2n+1}, kt) &\geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n}, t)\} \\ &\geq \min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t)\} \\ &= M(y_{2n-1}, y_{2n}, t). \end{aligned}$$

Similarly we put $x = x_{2n+2}$ and $y = x_{2n+1}$ in (4); we have

$$M(Lx_{2n+2}, Mx_{2n+1}, kt) \geq \min\{M(ABx_{2n+2}, STx_{2n+1}, t), M(Lx_{2n+2}, ABx_{2n+2}, t), M(Mx_{2n+1}, STx_{2n+1}, t), M(Lx_{2n+2}, STx_{2n+1}, t)\}$$

that is

$$M(y_{2n+2}, y_{2n+1}, kt) \geq \min\{M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n}, t)\}$$

$$M(y_{2n+2}, y_{2n+1}, kt) \geq \min\{M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n+2}, t)\} = M(y_n, y_{n+1}, t).$$

$$\begin{aligned} M(y_n, y_{n+1}, t) &\geq M(y_n, y_{n-1}, t/k) \geq M(y_n, y_{n-1}, t/k^2) \\ &\geq ..... \geq M(y_n, y_{n-1}, t/k^n) \to 1 \quad as \quad n \to \infty. \end{aligned}$$

So, $M(y_n, y_{n+1}, t) \to 1 \quad as \quad n \to \infty$ and for any $t > 0$. For each $\in > 0$ and each $t > 0$, we can choose $n_0 \in N$ such that $M(y_n, y_{n+1}, t) > 1- \in$. For $m, n \in N$, we suppose $m \geq n$. Then we have that

$$M(y_n, y_m, t) \geq \min\{M(y_n, y_{n+1}, \tfrac{t}{m-n}), M(y_{n+1}, y_{n+2}, \tfrac{t}{m-n}) ....M(y_{m-1}, y_m, \tfrac{t}{m-n})\}$$

$> \min\{(1-\epsilon), (1-\epsilon), ..., (1-\epsilon)\} \geq 1-\epsilon$, and hence $\{y_n\}$ is a Cauchy sequence in X.Since X is complete therefore $\{y_n\} \to z$ in X and its subsequences $\{ABx_{2n}\}$, $\{Mx_{2n+1}\}, \{STx_{2n+1}\}$ and $\{Lx_{2n}\}$ also converges to z.Since X is ϵ-chainable, there exists ϵ-chain from x_n to x_{n+1}, that is, there exists a finite sequence, $x_n = y_1, y_2$, ..., $y_l = x_{n+1}$ such that

$$M(y_i, y_{i-1}, t) > 1 - \epsilon, \quad for \ \ all \ \ t > 0 \quad and \quad i = 1, 2, ...$$

Thus, we have

$$M(x_n, x_{n+1}, t) \geq \min\{M(y_1, y_2, t/l), M(y_2, y_3, t/l), ...M(y_{l-1}, y_l, t/l)\}$$

$$> \min\{(1-\epsilon), (1-\epsilon), ..., (1-\epsilon)\} \geq (1-\epsilon).$$

For $m > n$,

$$M(x_n, x_m, t) \geq \min\{M(x_n, x_{n+1}, t/m-n), M(x_{n+1}, x_{n+2}, t/m-n), ... \\ M(x_{m-1}, x_m, t/m-n)\}$$

$$> \min\{(1-\epsilon), (1-\epsilon), ..., (1-\epsilon)\} \geq (1-\epsilon),$$

and so $\{x_n\}$ is a Cauchy sequence in X and hence there exists $x \in X$ such that $x_n \to z$. By the reciprocally continuity and Semi-compatibility of maps (L, AB) ; we have,

$$\lim_{n\to\infty} L(AB)x_{2n} = Lz, \quad \text{and} \quad \lim_{n\to\infty} AB(L)x_{2n} = ABz$$

and

$$\lim_{n\to\infty} L(AB)x_{2n} = ABz,$$

which implies that $Lz = ABz$.
Step (1): By putting $x = z, y = x_{2n+1}$ in (4), we get

$$M(Lx_{2n}, Mx_{2n+1}, kt) \geq \min\{M(ABz, STx_{2n+1}, t), M(Lz, ABz, t), \\ M(Mx_{2n+1}, STx_{2n+1}, t), M(Lz, STx_{2n+1}, t)\}$$

Letting $n \to \infty$; we get

$$M(Lz, z, kt) \geq \min\{M(Lz, z, t), M(Lz, Lz, t), M(z, z, t), M(Lz, z, t)\}$$

i. e.

$$M(Lz, z, kt) \geq M(Lz, z, t);$$

Thus we get $Lz = z = ABz$.
Step 2 : By putting $x = Bz$ and $y = x_{2n+1}$ in (4), we get

$$M(L(Bz), Mx_{2n+1}, kt) \geq \min\{M(AB(Bz), STx_{2n+1}, t), M(L(Bz), AB(Bz), t), \\ M(Mx_{2n+1}, STx_{2n+1}, t), M(L(Bz), STx_{2n+1}, t)\}$$

Since $AB = BA$, $LB = BL$; therefore $AB(Bz) = B(ABz) = Bz$ and $L(Bz) = B(Lz) = Bz$; Letting $n \to \infty$; we get

$$M(Bz, z, kt) \geq \min\{Bz, z, t), M(Bz, Bz, t), M(z, z, t), M(Bz, z, t)\}$$

i.e.

$$M(Bz, z, kt) \geq M(Bz, z, t).$$

Hence by Lemma 2.8

$$Lz = Az = Bz = z.$$

Since $L(X) \subseteq ST(X)$, there exists $u \in X$ such that $z = Lz = STu$.
Step 3 : By putting $x = x_{2n}, y = u$ in (4), we get

$$M(Lx_{2n}, Mu, kt) \geq \min\{M(ABx_{2n}, STu, t), M(Lx_{2n}, ABx_{2n}, t), \\ M(Mu, STu, t), M(Lx_{2n}, STu, t)\}$$

Letting $n \to \infty$; we get

$$M(z, Mu, kt) \geq \min\{M(z, z, t), M(z, z, t), M(Mu, z, t), M(z, z, t)\}$$

$$M(z, Mu, kt) \geq M(z, Mu, t).$$

$z = Mu = STu$. Since M is ST-absorbing then we have;

$$M(STu, STMu, t) \geq M(STu, Mu, t/R) = 1;$$

$$STMu = STu \Rightarrow z = STz.$$

Step 4 : By putting $x = x_{2n}, y = z$ in (4), we get

$$M(Lx_{2n}, Mz, kt) \geq \min\{M(ABx_{2n}, STz, t), M(Lx_{2n}, ABx_{2n}, t), M(Mz, STz, t), M(Lx_{2n}, STz, t)\}$$

Letting $n \to \infty$; we get

$$M(z, Mz, kt) \geq \min\{M(z, z, t), M(z, z, t), M(Mz, z, t), M(z, z, t)\}$$

$$M(z, Mz, kt) \geq M(z, Mz, t).$$

Hence by Lemma 2.8 z = Mz = STz.
Step 5 : By putting $x = x_{2n}, y = Tz$ in (4), we get

$$M(Lx_{2n}, M(Tz), kt) \geq \min\{M(ABx_{2n}, ST(Tz), t), M(Lx_{2n}, ABx_{2n}, t), M(M(Tz), ST(Tz), t), M(Lx_{2n}, ST(Tz), t)\}$$

Since $ST = TS$, and $MT = TM$; therefore $M(Tz) = T(Mz) = Tz$, $ST(Tz) = T(STz) = Tz$ and Letting $n \to \infty$; we get

$$M(z, Tz, kt) \geq \min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t), M(z, Tz, t)\}$$

$$M(z, Tz, kt) \geq M(z, Tz, t).$$

Hence by Lemma 2.8 z = Tz = Mz = Sz. Therefore $z = Az = Bz = Sz = Tz = Lz = Mz$. i.e. z is a fixed point of A, B, S, T, L and M.
Uniqueness : Let w be another fixed point of A, B, S, T, L and M ; therefore putting $x = z$ and $y = w$ in (4), we have

$$M(Lz, Mw, kt) \geq \min\{M(ABz, STw, t), M(Lz, ABz, t), M(Mw, STw, t), M(Lz, STw, t)\}$$

$$M(z, w, kt) \geq \min\{M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t)\}$$

i. e., $z = w$ by Lemma 2.8 . Hence z is a unique fixed point in X. This completes the proof. □

Corollary 3.2. Let A, B, S, T, L and M be self mappings of a complete ϵ-chainable fuzzy metric space $(X, M, *)$ with continuous $t - norm$ defined by $a * b = \min\{a, b\}$, satisfying (1) - (4) of Theorem 3.1 and there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t), M(Lx, STy, t), M(ABx, My, 2t)\}$$

If $\{L, AB\}$ is reciprocally continuous semi-compatible maps. Then A, B, S, T, L and M have a unique fixed point in X.

Proof. Since

$$M(Lx, My, kt) \geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t), \\ M(Lx, STy, t), M(ABx, My, 2t)\}$$

$$\geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t), M(Lx, STy, t), \\ M(ABx, STy, t), M(STy, My, t)\}$$

$$\geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(My, STy, t), M(Lx, STy, t)\}$$

and hence, from Theorem 3.1, A, B, S, T, L and M have a unique fixed point in X. □

Corollary 3.3. Let A, B, S, T, L and M be self mappings of a complete ϵ-chainable fuzzy metric space $(X, M, *)$ with continuous $t - norm$ defined by $a * b = \min\{a, b\}$, satisfying of Theorem 3.1 and there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Lx, My, kt) \geq M(ABx, STy, t)$$

If $\{L, AB\}$ is reciprocally continuous semi-compatible maps. Then A, B, S, T, L and M have a unique fixed point in X.

Proof. Since

$$M(Lx, My, kt) \geq \{M(ABx, STy, t), 1\}$$

$$\geq \min\{M(ABx, STy, t), M(Lx, Lx, 5t)\}$$

$$\geq \min\{M(ABx, STy, t), M(Lx, ABx, t), M(ABx, Qy, 2t), \\ M(Qy, STy, t), M(STy, Lx, t)\}$$

and hence from Corollary 3.2, A, B, S, T, L and M have a unique fixed point in X. □

Let AB and ST be the identity mapping on X in Corollary 3.3. Then we get the next result.

Corollary 3.4. Let L and M be self mappings of a complete ϵ-chainable fuzzy metric space $(X, M, *)$ with continuous $t - norm$ defined by $a * b = \min\{a, b\}$, satisfying the following condition ; there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Lx, My, kt) \geq M(x, y, t)$$

Then L and M have a unique fixed point in X.

In Corollary 3.4, if we take $L = M$, then this result become to Banach contraction theorem.

Corollary 3.5. Let L be self mappings of a complete ϵ-chainable fuzzy metric space $(X, M, *)$ with continuous $t - norm$ defined by $a * b = \min\{a, b\}$, satisfying the following condition ; there exists $k \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Lx, Ly, kt) \geq M(x, y, t)$$

Then L has a unique fixed point in X.

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